CHANGING CONCEPTIONS OF STATISTICS; A PROBLEM AREA FOR TEACHER EDUCATION

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1. Changing conceptions of statistics

Implicit or explicit answers to the question "what is statistics" have been highly variable in history. They are nevertheless very important in curriculum research and design and in teacher education. Let us look for first approximations. Gnanadesikan and Kettenring (1988, p. 13) characterise statistics as a "data science" with close synergetic relations to mathematics and computing science. This is in fact a very modern definition which brings the changed nature of statistics to the point; putting "data" at the centre of statistics and mentioning computing science on an equal standing with mathematics as a closely related discipline. In the circumstances, it is important to remember, however, that it is more than 25 years since John Tukey (1962) wrote a programmatic article on "The future of data analysis" where he used the notion of data analysis as a challenge to a traditional interpretation of statistics and with the aim of putting "data" in the centre.

For further clarification it helps to distinguish the practice of analysing data as one level of statistical activity from the level of constructing, comparing and optimising tools for statistical practice. In other words, the "practical perspective" has to be distinguished from the "theoretical" one or the "perspective of the scientific discipline". It is quite clear that changes have been dramatic on both levels. At the disciplinary level, there were times where a theory ideal of mathematics was dominant. Today, a really new emphasis on tool construction and design, i.e. on the engineering or technological aspects of disciplinary statistics can be observed. This shows clearly when we consider the development of statistical software tools an essential part of the discipline of statistics.

Only a small minority of mathematics teachers who are supposed to teach statistics as part of general mathematical education at secondary level can be expected to have experienced statistics from the above point of view and to hold such views and attitudes on statistics. This is particularly true for the close relation to computing on a practical and discipline level and to putting "data" at the centre of statistics instead of

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emphasising statistics' relations to probability or regarding "sample to population inference" as essential characteristic of statistics.

Usually, "data" are not well received parts of mathematics and mathematics curricula. With relation to the synergetics of mathematics in general with computing, however, there are indications of change. I think it will be quite helpful to regard the further development of school mathematics as a development from "mathematics" to the "mathematical sciences" (see Usiskin 1988). From this perspective, a "modern" characterisation and realisation of statistics for teacher education and for the classroom may become one important moment in this transformational process.

2. Metaknowledge as part of teachers' professional knowledge

2.1. The relevance of metaknowledge

What is statistical knowledge? This question has become quite relevant in designing statistical expert systems. Probably we will know more about the structure, the components and the types of knowledge required of a statistical expert in the future. In any case, the question of statistical knowledge is highly relevant for statistical education. It has often been raised when objecting to teaching only an idealised or short cut process of statistical inference. Instead, many people have been looking for guidelines and principles which can orientate practical statistical work and which express self-interpretations of statistics.

We will use the notion of metaknowledge, i.e. knowledge about statistics as follows. Metaknowledge consists of the partly tacit and eclectic epistemological and philosophical self-interpretations and orientations of statistics. They are (only) partly drawn from closed "philosophies of inference" and regulate, for instance, the application of statistical knowledge and tools in practice, the cooperation with subject matter experts and the construction and evaluation of software and other tools of the scientific discipline of statistics. The relevance of metaknowledge in statistical practice is shown, for instance, by the controversies between Bayesian and non-Bayesian statisticians.

Although the borderline between statistical knowledge and metaknowledge is variable, the distinction is important for purposes of clarification. For instance, a distinction between Bayesian methods and Bayesian metaknowledge can be made. Tukey (1972, 1962) uses a similar distinction, rejecting important ingredients of Bayesian metaknowledge, arguing at the same time for a limited and situation-specific use of Bayesian methods in an exploratory spirit (metaknowledge).

The same distinction applies to classical techniques. During the beginnings of developing Exploratory Data Analysis (EDA), for instance, it was argued that standard tools like ANOVA or linear regression can be used in a classical (confirmatory) frame or in an exploratory frame or spirit. In the classical frame, the things which matter are estimation of parameters, confidence intervals and significance testing, whereas the same tools can be used for summarising data and for generating residuals, which may give interesting clues for further exploration. This differentiation on the level of metaknowledge is obviously most important from a practical perspective. It is of similar importance for the perspective of the scientific discipline as it will also guide the construction and design of tools. (See Biehler 1982, p. 168, for the use of the notion of metaknowledge for analysing the development of EDA.) In controversial discussions on EDA, metaknowledge plays a major role, controversies on single techniques are less important.

Gnanadesikan and Kettenring (1988, p. 14) argue that teachers of statistics should convey the "pragmatic tendency to use whatever statistical thinking and techniques that are appropriate for the problem at hand, without being tied to a particular approach or school of thought". This attitude does not, however, imply the irrelevance of metaknowledge. On the contrary, it calls for a even greater consciousness of metaknowledge because different styles of statistics have to be distinguished, related, selectively explained and applied. This will make teaching more difficult.

2.2. Metaknowledge for teachers

The general need for metaknowledge, for knowledge about mathematics, as part of teachers' professional knowledge, has often been worked out (see e.g. Arbeitsgruppe Mathematiklehrerbildung 1981, Otte 1979). Steinbring (1988) reports on a project where the notion of metaknowledge in probability and statistics was a leading concept for in-service training and the design of classroom material. But it is still an open question what detailed components and structure "metaknowledge" for teachers in statistics should have and by which means it could be made accessible to teachers in particular when we take recent changes in statistics into account.

We often hear the complaint that teachers should have had more experience with the analysis of real data, preferably with the support of interactive statistical software. See, for example, the forum in the College Mathematics Journal, Vol. 19.1, (1988). Although this demand is quite understandable given the background of current teachers of statistics, the implicit underlying idea that the teacher should become a practical statistical expert "en miniature" has to be carefully considered. The question should be reformulated as to what may be differences and similarities concerning the metaknowledge of teachers of statistics on the one hand, and of statistical experts on the other. It may well be true that similarities increase if we regard the statistical expert not only as a person working with real data, but also as a person working with real people to whom he has to present and explain the results and the choice of tools and with whom he is involved in complex networks of interests and other social boundary conditions.

Basically, the difference in experience and knowledge between a statistical or applied mathematical expert and a teacher in general education is inevitable, although it may be helpful to bridge the gap through project work in industry for teachers, (see Eyre 1988). A remark by Freund in the above mentioned forum about how one can get a feeling for statistics goes in a similar direction.

"We may get this understanding of what is really involved through lots of hands on experience working with data, or we may get it through intellectual pursuits, such as through reading, listening to those who are knowledgeable, and giving the matter a great deal of thought. Most fruitful, perhaps, would be a combination of the two." (Freund 1988, p.18)

To this, we may add the truism that experience alone is a very poor guide. The widespread misuse or poor use of statistics gives additional support to this claim.

In addition, the situation of teachers in secondary school requires other types of metaknowledge. This certainly should include a historical, cultural and epistemological perspective on statistics and its relations to other domains of science and practice as well as relevant basic philosophical ideas. We may call this an "extended perspective of the discipline", which is different from the practical perspective and the related metaknowledge of practitioners.

The traditions of school mathematics form an important boundary condition of any innovation, and metaknowledge on statistics has to be critically related to this situation. This problem is currently being explored in a joint project with Heinz Steinbring at our institute in the GRAPHDAS project (Graphs for Data Analysis at Secondary Level). We are exploring how teachers react to ideas of Exploratory Data Analysis with the intention of learning more about implicit views on mathematics and conditions for changing them. An important dimension of the in-service training component of GRAPHDAS consists of metaknowledge which is to regulate the teachers attitudes and reflections about teaching experiments which they are supposed to do on elementary concepts and tools of data analysis. The project is based on two major sources; firstly on an analysis of the problems of integrating probability and statistics into traditional school mathematics and principles for cooperative work with teachers in connection with in-service training, which is described by Steinbring (1988); and, secondly, on a detailed analysis of EDA and its controversial development (Biehler 1982, Biehler 1985). In the following, I shall describe some basic current ideas concerning the content of metaknowledge on EDA for teachers.

3. Elements of teacher knowledge about developments related to Exploratory Data Analysis

The following ideas describe the starting point in the GRAPHDAS project. They will certainly benefit from critical discussion and further experience in working with teachers. The type and content of metaknowledge teachers should acquire is certainly dependent on the intended content of teaching. The realm of tools and concepts we regard as potentially relevant for secondary level and the aims of the project can be summarised in the following three groups;

Tool/representation	Concepts
table, stem-and-leaf, dot-plot, histogram, bar chart	variation, symmetry, skewness, separation into groups, outlier
5(7)-number summary box-plot	median/quantiles/extremes, resistance summarising and unfolding data, comparison of data sets
scatterplot; - lineplot - several variables in one graph - scatterplots with summaries (lines, curves)	data transformation, change of scale data=model+residual eye fitting lines slicing and median curve 3 group line (resistant Tukey line)

Three areas of metaknowledge concerning critical and new aspects of EDA were considered important ;

* data and their context
* activity of data analysis
* graphs

3.1 Data and their context

The keywords we used concerning data and their context are

* multivariate perspective
* open context
* developing interpretations
* structures/patterns and anomalies

An important background for understanding EDA and other new developments is a "multivariate perspective", even if only univariate or bivariate "techniques" may be relevant for secondary level. Most secondary curricula on probability and statistics concentrate on one-dimension al data with random variables. If two-dimensional problems are considered at all, they receive a rather technical treatment, and two-dimension al data mostly appear as if they belonged to a completely different ontological category than one-dimension al data. One of the reasons for this is the implicit orientation towards probability and inferential statistics which can hardly be developed beyond the univariate level at school. Historically, this implicit belief was also a major obstacle for the development of new multivariate methods within the prevailing tradition of statistics.

For establishing a multivariate perspective, it may be quite helpful that teachers get experience with exploring rich data bases with the support of interactive computer software. Using simple statistical tools like numerical summaries, box-plots, scatterplots and bar charts, but combining them with the power of a software environment which supports the selection of subsets of data, transformation of variables and creating new variables in a spreadsheet-like manner may already provide a powerful experience.

Such experience seems to be important, even if comparable computing power is not yet available at school level. The multivariate perspective sheds also new light and places more pedagogical emphasis on the many data sets in newspapers and consumer magazines which all are multivariate. Stem-and-leaf plots and box-plots should also be used in a multivariate flavour because they can be applied to comparisons of data batches or to opening first insights into the underlying multivariate system. Also, individual values can often be particularly represented and may indicate the potential influence of further variables, (see Biehler 1988).

A key problem is the relation between data and their context, the problem of meaning and interpretation. The key notions "open context" and "developing interpretations" address this problem. Teachers who are willing to try out more practical and applied approaches to statistics usually intend to overcome the situation that data are merely considered as numbers and as input for a certain technique. They emphasise, for instance,

* a careful consideration of what the numbers mean/refer to, how they were measured or constructed from directly measured quantities.

* that techniques should be chosen according to subject matter problems and objectives. Interpretations should be guided by subject matter knowledge and take into account that data are only a "model" of the situation.

* that data should be collected or selected according to a subject matter problem.

EDA is often reinterpreted from this perspective. Concrete material which aims at or which could be used in teacher education partly emphasises these aspects, e.g. Landwehr/Watkins 1986, Borovcnik/Ossimitz 1987, Statistics in Society 1983, Graham 1987. Nevertheless there are certain tensions with the orientations that underlie Tukey's basic book on EDA (1977). J.A. Nelder (1985, p. 238) brings a basic criticism of this book to the point when commenting on Chatfield's (1985) critical evaluation of Tukey's EDA.

> "From probability theory with no data, its [EDA's] proponents have swung to data without any models, and the methods they encourage are in danger of falling into mere playing with numbers. Who wants to look at the height of the world's 100 highest volcanoes, considered as a set of 100 numbers, ignoring entirely the existing corpus of volcanology? This seems to me as pointless as doing a significance test on a null hypothesis known to be false."

Although the temptation of "merely" playing with numbers is very real, especially in flexible computer environments, and although it may be that reading Tukey's (1977) book will also have negative effects in this direction, Nelder's criticism has to be put into perspective. First of all, it has to be underlined that a somewhat experimental "playing with numbers", although not a "mere" playing, has become a really new possibility in statistical practice. This possibility gives data a fundamentally new role and opens other approaches to data than were possible before, when straightforward and pre-planned analyses were predominant. These new features, in general, make data analysis more difficult, perhaps more frustrating and not easier than classical approaches, contrary to what is often assumed. Secondly, as has been argued elsewhere (Biehler 1982), Tukey's book can be reinterpreted as promoting a new perspective of the scientific discipline on data analysis, i.e. emphasising what can be done from the perspective that "the data are given". This explains an underlying emphasis in Tukey's book, namely that "playing with data" according to relative abstract principles such as looking for outliers may contribute a new perspective on a subject matter problem. Problems will arise when the relative perspective of the book is made absolute and interpreted as providing "complete" orientations for practical work with data.

For practice, it seems to be most fertile combining both perspectives; that the context should influence the data analysis and that the data analysis should influence the context.

This harmonising attitude, however, may conceal the new possibilities of Tukey's basic approach to EDA in a pedagogical context where expert knowledge in a subject matter domain cannot be assumed. For instance, a small table of suicide rates for one year and different age groups, sexes and countries may be explored with a number of interesting discoveries and without assuming *a priori* knowledge of the sociology of suicide (see e.g. Biehler 1988). As a result of such an exploration, students may know a bit more about possible variables that are related to "suicide rates" and may also be more competent in asking experts differentiated questions. They may also become more interested in looking up further data or investigating how the various rates were measured than they were at the beginning of the analysis (open context). In other words, the exploration of data can be used to learn more about a certain subject matter and to enable students to ask more sophisticated questions in this area, (developing interpretations).

We have to go one step further. From a developmental point of view, it is quite clear that the kind of "subject matter questions" that people will and can ask are also dependent on the elaboration and availability of their statistical concepts and tools. If we take a table with the number of traffic accidents for each day of the year, as an example, questions may develop from "what is the day with most accidents?" to "how does the distribution of accidents vary with the month of the year, especially with regard to outliers, spread and level?". In summary, from a developmental point of view, the elaboration of the students' repertoire of data analytic or statistical concepts, tools and representations is important and not merely the problem of meaning and interpretation in a single application.

The overall objective of EDA can be characterised as looking for patterns/structures and anomalies in data. As this innocent characterisation is probably also true for other areas of science and statistics, it deserves further specification. This demand is particularly important for teachers in secondary education who have to be aware of different tacit orientations of various school subjects and within statistics. Let us discuss an example which is to illustrate different approaches to "general law vs. structures in single data sets", of detecting "significant" and other anomalies, of modelling variation by probability models vs. displaying residuals for further analysis and interpretation.

In Graham (1987, p. 41), a book influenced by ideas from EDA, we find the suggestion that students should make measurements of the diameter and the circumference of several circular objects. The data are plotted in a scattergram. The main discovery is the linear structure in the plot, i.e. the approximate proportionality. A fitted line could reasonably be used for extrapolation and interpolation. We can also give a theoretical explanation for the approximate linear relationship. Additionally, unlimited experimentation is possible. I would consider this as a completely traditional framework for using graph paper in science to discover and describe scientific laws. A further step in sophistication would be to model the deviations from the main pattern (the line) by a probability model. The possibility of reproducing variation patterns in real data, the possibility of predictions in uncertain situations by theoretical models was quite an achievement in the history of science. It will become salient when we simulate such a situation and plot the simulated data in a scatterplot, which will look "very similar" to the original one. As a next step of sophistication, inferential statistics has been designed for, roughly speaking, detecting "anomalies" which cannot be explained by random variation alone.

What would be a typical modification of the above example illustrating EDA's approach? Looking at the residuals and trying to analyse and interpret them would be an option. Maybe gross measurement errors could be detected and corrected by repeating a measurement, maybe the influence of different measurement techniques could be seen. This would already be one step away from an exclusive concentration on the basic (deterministic) law. Now, imagine a collection of circles and squares made of cardboard of slightly varying thickness. Students get a data set of measurements of diameter and weight of this collection and should explore a scatterplot of the data with the objective of finding out something about characteristics of the collection. In this case, the analysis of a single data set is already at least equally important as the question of general law. In this case, law-like relationships can be used to analyse single data sets and their specific characteristics. If we go even farther in this direction, we will find the use of lines in scatterplots as a visual help for analysis or as a summarising description of a single data set, together with severe warnings against any ontological claims for the line as a law or as a unquestioned basis for extrapolation, interpolation and prediction (Tukey 1977, p. 180, see also Biehler 1982, p. 319).

Some of these differences are present in existing materials for students and teachers. In Landwehr/Watkins (1986, p. 108), for instance, lines on scatterplots are introduced because "interpretations of scatterplots are sometimes helped by adding a line". Later, lines on scatter plots are sometimes used as if they represented a generalisable law, sometimes with merely descriptive purposes, sometimes as a reference frame for depicting extraordinary points. No example, however, is given where a theoretical explanation of a linear relationship could be furnished.

I think that teachers would need a much deeper and differentiated understanding of these aspects of patterns and anomalies than is currently given in textbooks for students. Otherwise teaching EDA may become something completely separated from the rest of school education.

3.2. Activity of data analysis

It is impossible to discuss data analysis without discussing the activity of data analysis. In discussions with teachers, the (mathematical) simplicity of some of the new techniques is often emphasised. It can be argued, however, that the intended style of their use is generally much more difficult and complex than a routine application of standard techniques. This diagnosis is well reflected in recent attempts to enhance data analysis software with "intelligent guidance" components because non-expert users are daunted.

When we intend to explore the pedagogical potential of the new style of activity, a one-sided emphasis on "simplicity" may be the wrong policy.

The following notions shall provide a basic orientation for problem areas;

* data analysis is detective work with data
* interactive, experimental
* openness and flexibility
* multiplicity and uncertainty of results

Tukey's metaphor of detective work emphasises the role of discovery, but also the complexity of the data analytic activity. It relates to the stage of data analysis and not necessarily to the stages of data collection and interpretation, which can also have detective-like characteristics. Interactivity implies a multi-stage process where each new step is dependent on a decision for which the result of the preceding step provides an important input. Decisions may concern what to look for next, which tool to apply, which part of the data to use, which new data to collect, which body of subject-matter, knowledge to study, etc. Experimental style implies an attitude of trying out methods or tools without being able to make it highly probable in advance that the method will be helpful. Openness and flexibility refer to the attitude and ability to react to new situations and conditions in the course of analysis. For instance, when we compare several data sets with box-plots and see that one batch is separated in two distinct groups, we may decide to use two box-plots for this batch. However, no general rules can be formulated and much depends on the concrete situational context.

Teachers should become able to regard the decision points as opportunities for discussion, social learning and specific stimuli for students' cognitive development.

A further basic problem is the character of results of data analysis, their subjectivity, multiplicity and uncertainty. This is much debated in statistical science. For instance, some argue that exploratory results can have only a limited value in themselves but have to be complemented or critically examined with confirmatory methods. Diaconis (1985) develops a broader critical perspective on EDA attacking the question of what means and measures may help controlling the partly "magical thinking" practised with EDA. He summarises a spectrum of remedies, among others; making the exploratory character explicit, trying out on fresh data, borrowing strength from similar situations, cross-validation and bootstrapping. Additionally, results from data analysis can also be criticised from a subject matter point of view, i.e. it can be required that they have to be discussed against the existing body of subject matter knowledge (see section 3.1). This can be extended toward a fundamental confinement of EDA's principle of "looking at the data" emphasising the role of theory building and model construction instead.

Many people who apply statistical methods are not aware of these controversies. How much should teachers of statistics know about these problems ?

In the context of school teaching, these problems have an additional dimension. Scientists usually have another relation to knowledge than is encouraged in schools. Scientific knowledge is considered to be much more variable, uncertain, preliminary, debatable and "under development" than school knowledge. School knowledge is usually considered to be a carefully selected part of what is really known in science, equipped with much more definiteness and authoritative back-up. A most fertile context for teaching ideas from EDA would be when teachers adopt the "knowledge under development" point of view. This could have the pedagogical effect of learning to mistrust all the "definite conclusions" which are publicly offered.

3.3. Graphs

It is helpful to distinguish the following perspectives on graphs;

* functional or methodological perspective; goals and principles for using graphs (in statistics).

* cognitive or psychological perspective; cognitive difficulties, misinterpretations.

* designer's perspective; how to make "good graphs".

* historical and cultural perspective; changes in using statistical graphs and possible explanations for these changes.

Insights and information resulting from those perspectives have to be combined. (See also Vännman 1988 for an approach to statistical graphs in teacher education.) In our project, leading ideas have been formulated in abstract terms to facilitate a transfer to non-statistical uses of graphs;

(1) Graphs are used as exploratory tools, to develop new knowledge and insight, discover new things about data and their context.

(2) Graphs are used as relational diagrams, i.e. the goal for using graphs is to portray and make accessible relations in data.

(3) Graphs have the character of a model, i.e. they may be more sensitive to some relations than to others or they may show only parts of the whole data set (looking through a "window" on the data), etc. In other words; they define a perspective on the data, and do not show the data "as they are".

(4) Varying graphs and using a multiplicity of graphs is an important "new" method to extend the uses of graphs.

It is important to relate these ideas to other experience and attitudes teachers have on graphical representation. Particularly, a critical attitude toward the wide-spread opinion that graphs are primarily pedagogical tools to make things easier for students has to be conveyed.

Classical diagrams such as histogram, pie chart, bar chart, table can be re-experienced as exploratory tools if they are compared to an ordered or unordered list of data with regard to the possibilities of discovering relations. The invention of the histogram, i.e. the idea of using "discrete" bar charts to represent continuously varying data, was quite an achievement in history. If compared to a dot-plot, a histogram can be re-experienced as an exploratory tool for revealing structure in data, (concerning the form of the distribution). Some may also rediscover the table as a semi-graphical exploratory tool.

The use of statistical graphs in mass media is often conceived of as a challenge for statistical education. This attitude is adopted by teachers who regard the education of critical consumers of statistics to be the major objective of statistical education. Huff's (1954) book is often quoted in materials for teacher education. The limited and somewhat negative attitude towards graphs which often results from adopting this perspective is hardly compatible with the new appreciation of graphs in scientific statistics. The new perspective on graphs, however, can also be used to convey a deeper understanding of the "Huff tradition".

For instance, Wainer (1984) presents a collection of mass media graphs under the heading of "how to display data badly". He shows why they may be called "bad" and how a better graph could be designed. Although his article can be used as a valuable resource, its general orientation of "improving graphs" seems to be one-sided if it is not complemented by the appreciation of the model character of graphs. Decoration of graphs, deception with graphs, and graphs that are poorly designed with regard to perceptual abilities of readers are problems in mass media which teachers should know about. But, it is at least as important to experience that a graph can be "improved" in many directions, in other words that each improvement may focus on somewhat different relations. Thus, as a consequence of the model character, a multiplicity of complementary graphs will be helpful. In addition, the graphical display of data with more than two dimensions is a really big problem and choosing a "window" or "projection" of the data is inevitable. The deceptive exploitation of this feature should be viewed in this broader context.

Operating on graphs, varying and modifying them, using a multiplicity of graphs is an important possibility which has become really practical only with modern computing equipment. The possibility of multiple, dynamically linked representations is sometimes classified among the most important impacts of computing on mathematics education, (see Kaput 1986). Teachers may have experienced this feature in other mathematical domains like the graphical exploration of functions. In this classical curriculum area, however, the new possibilities of multiple graphs cause new problems on the learners' side which have to be carefully taken into account, (see Goldenberg 1988). In any case, EDA's perspective on graphs regards them as complementary to analytical methods and as having specific, non-substitutable functions for cognition and communication. This seems to be a more adequate general perspective on graphs than merely regarding them as simplifying pedagogical tools.

With regard to statistics, the multiplicity of graphs contradicts the traditional principle in descriptive statistics of "reducing information" as a main goal and therefore it is important to underline the complementary principle of unfolding data. The danger of undigested or indigestible volumes of output has to be confronted with the danger of straightforward and standardised procedures of "reducing data".

4. Concluding remarks

As pointed out above, the ideas of section 3 are in a stage of development. Their further development needs more experience and research. We started the GRAPHDAS project with about 15 teachers and used various means for developing basic ideas such as

* lectures and material on the "philosophy" of EDA,
* techniques and their application,
* examples from practice,
* cooperative work at the computer (for instance with STATVIEW 512+),
* discussing initial classroom experience of some of the teachers.

One of the major findings was, of course, the need for more time. The compressed experience at the computer with highly interactive and graphical statistical software was valuable to get an overall impression of statistical work. On the other hand, the extensive discussion of a very few paper and pencil examples had a complementary function and reduced the problem of short-cut interpretations of the many graphs at the computer.

In general, teachers seemed to find the metaknowledge dimension helpful. They used the key notions and generated their own interpretations related to their classroom practice and experience. This context of interpretation was more dominant than expected. But it was just those teachers who had specific initial experience with teaching ideas from EDA and not only of other approaches to statistics who felt a greater need and who were better motivated for general orientations. A further experience was the fragility of new metaknowledge, which became visible in follow-up cooperative work on planning the teaching unit after the initial seminars. This supports the necessity of continuing cooperation.

Supplement to Rolf Biehler's presentation

Andee Rubin: I have two questions about the last paragraph of your paper. First, you say that individual teachers' "context of interpretation was more dominant than expected". Could you explain what you mean by "context" and why you were surprised by its dominance?

Second, you say that teachers who had initial experience with EDA were more open to your perspective. Did you find teachers for whom previous experience, for example with the school mathematics culture, was an interference?

Rolf Biehler: We started with the idea of bringing the statistical experiences closer to the teacher, by summarising key points, providing opportunities for using the computer, and so on, and then going on to discuss the implications of these ideas for classroom practice. But we found even at the first session that the teachers themselves tried to move away from the material we had hoped might give them some characteristics of data analysis, towards consideration of the materials for classroom practice. This was the context of interpretation which became dominant.

In answer to your second point, we found that people who already had some experience of EDA were interested in questions of interpretation and proper use of the techniques, whereas the others were mainly interested in understanding what the new ideas were and their possible uses in a teaching context.

David Vere-Jones: Could I ask how teachers were selected to take part in the activities you have described?

Rolf Biehler: They were a rather special group of teachers from the comprehensive schools, most of whom we knew well, and with whom we had worked for several years. Also, and this is rather a special feature which is relevant to the previous question, these teachers were strong proponents of applied mathematics, critical of traditional mathematics, and keen on getting the ideas of probability and statistics accepted. Moreover, they had previously been teaching statistics using the data from surveys and opinion polls collected by the students themselves, so they were against trying any analysis on the data without knowing exactly how the data had been collected, and so on. In practice, however, we found that the initial EDA prompted the students to ask questions about what the data meant and how it had been collected and as a result the teachers changed their views, at least to some extent.

Y.L. Cheung: I agree with you that a multivariate perspective is a useful concept for the understanding of EDA and even for classical statistics. But the idea of multivariate analysis is much more difficult for schoolchildren to understand. To what extent is this idea being taught in schools in your country, and how successful is it?

Rolf Biehler: I deliberately said "perspective", not "methods", to indicate that the aim was not to teach what we commonly describe as multivariate methods but only to give teachers some insight into the use of simplest bivariate techniques such as scatterplots, box-plots, etc., to explore data from a multivariate data set. For example, we studied in this way the number of traffic deaths in relation to a series of other variables such as times of the year, days of the week, and so on, making use of the powerful EDA software to select suitable subsets of the data for display.

Kerstin Vännman: According to my experience from in-service training of teachers, the stem-and-leaf displays and the box-plots are very useful to help teachers who are a bit afraid of statistics to start to work with data and statistics. What is your experience in that respect?

Rolf Biehler: Our teachers were not so much afraid of statistics, but rather objected to box-plots and stem-and-leaf diagrams as being too advanced techniques. We had said that box-plots and stem-and-leaf plots could be useful for comparing distributions, and they replied that comparing distributions was a very advanced topic lying outside the school course and so these techniques would not be suitable. However, the teachers who tried it found that this was not true for the students, especially the stem-and-leaf, because it enabled the idea of the distribution to evolve naturally from the data without technicalities of class intervals, etc., required for drawing a histogram. Even more convincing for the teachers was that for the first time their students now had some "hands-on" techniques they could use for their own purposes.

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