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CONSTRUCTING STOCHASTIC SIMULATIONS WITH A COMPUTER TOOL - STUDENTS' COMPETENCIES AND DIFFICULTIES

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Constructing stochastic simulation with the computer tool Fathom™ has become an important part of an elementary stochastic course for student teachers at our university. We developed a three-step-design with a probabilistic part and activities with the software. Using a schema of the simulation process and different Fathom competencies we analyzed data from videotapes and semi-structured follow-up interviews. We sought to elaborate the problem solving process of students working on a simulation task focussing on how the students acquired Fathom and mathematical competencies. Some conclusions are that the competencies required by the students in different depths and the connection of Fathom objects to mathematical concepts have to be established in teaching.

INTRODUCTION

Simulation can serve two pedagogical purposes: Simulation can be used as a tool to solve problems and to make random situations more experiential (Biehler, 1991). Simulation is a mathematically more elementary method than calculating, so students can solve problems that are not otherwise solvable for them in a theoretical way. Also simulation can be used in combination or instead of analytical or combinatorial methods. We intend to use simulated models as experimental environments to support meaning construction in stochastic situations with various concepts like probability, event, random variable, or expected value. Another purpose for using simulation is to build up probabilistic intuitions.

At our university the tool-software Fathom is continuously used to support the learning and working processes in an elementary stochastic course for student teachers for grades 5 to 10 (pupils' ages are 11 to 16 years). The software is introduced and used for exploratory data analysis and descriptive statistics in the first part of the course. The students can use the learned software capabilities to analyze simulated data later on. The second part of the course concerns elementary probability. An important topic in this part is the simulation of random experiments, like the approach of Konold (1994) to estimate probabilities through simulation in introductory probability courses. The simulation in Fathom is introduced in parallel to the concepts of probability, random variables and events. Random situations are to be modelled mathematically and simulated; both results are to be compared if available.

The software is used as a student tool for actively analyzing data, simulating and building models as well as for exploring methods and concepts. During the whole term the students have to work with this software. The students are required to express the probability model in mathematical language, as well as using the language of events and random variables for explicitly expressing assumptions. The same requirements hold for student activities with Fathom. They have to construct a probability model in Fathom and they are to use the "Fathom-language" for defining events and random variables. Also, students have to use Fathom as a data analysis tool and to document their work and results.

THE STUDY

This study is part of a research program concerning teaching experiments with Fathom in elementary stochastic courses at our university as well as in "standard stochastic courses" at upper secondary levels at several schools (Biehler, 2003). Several studies aim to explore the relationship between students' stochastic thinking and knowledge and their simulation activities in Fathom in different ways (Biehler, 2006). Here we present some results of a case study with eight student teachers, who were asked to solve a simulation task with Fathom in pairs. We videotaped the working process and communication of the students and captured their computer activities with a screen capture program. Subsequently we watched the recorded material with every single student in individual sessions with a "method-mix" of stimulated recall and half structured interview.

Hypotheses of the study are based on three essential support features of the software Fathom for constructing simulations. One capability of Fathom allows students to construct and represent probabilistic models by various random machines. The random machines act for different kinds of random experiments. The second capability is the use of Fathom as a simulation tool itself. Theoretical probabilities can be estimated through relative frequencies and distributions of random variables through their empirical distributions. And third Fathom offers the possibility to experiment with the model.

To use Fathom for modeling, simulating and experimenting the students need certain Fathom competencies. Two questions of our research are therefore: Which Fathom competencies do students need? And to what extent did the students in our study acquire Fathom competencies to solve a typical task? A second field of research concerns the fact, that simulation with Fathom allows students to solve problems they would not be able to solve in a theoretical way, because in simulation mathematical competence is substituted by Fathom competence. Will thereby students more easily concentrate on probabilistic aspects during their problem solving process? Fathom reifies the concepts of event and random variable by means of the Fathom-object of a "measure": A random experiment is represented in Fathom as an attribute whose values are generated by a random machine (a random command). A “measure” is a function that can be defined on the results (the attribute values) of a random experiment. We also have the hypothesis that use of Fathom supports the use of these concepts as empirical concepts in the modelling context. But a transfer from empirical to theoretical terms probably works only with an additional theoretical treatment in the stochastic course (see the Summary and Consequences).

IDEAL WORKING PROCESS FOR SIMULATIONS WITH FATHOM - STAGES AND DECISIONS

In this section, probabilistic and simulation steps and decisions are described that take place in a normative task treatment. In a prior didactical analysis of the simulation capabilities of Fathom, we developed concepts and notions to use the software as a simulation tool. We illustrate the conceptualized activities first in an abstract way and second by a concrete task, and also show potential deviations from the ideal pattern and sources of problems. The following problem analysis is partly based on research results and on a theoretical analysis (Maxara & Biehler, 2006).

We envisage that students would work on simulation problems in three steps: setting up a stochastic model of the random situation by using probabilistic concepts, writing a plan of simulation and realizing the plan in Fathom. These three steps would be used as a modeling guideline for simulation.

In the first step the students are expected to model the random situation by building up a model of a real situation - a "model random experiment". They would describe it in a concrete model, for instance by an urn-model. They would construct the model, to identify the sample space, the probability distribution, as well as the events and random variables of interest. In the second step the students would transform the probabilistic concepts into a simulation plan. They could do this transformation step by step, perhaps as we exemplify it below.

For students' orientation we developed a four-step-design in two columns: probabilistic concepts and Fathom objects and operations, which correspond to each other in each step. We call the concrete description of the four stages in Fathom (right column) the "plan of simulation", built on ideas by Gnanadesikan, Scheaffer et al. (1987). The pedagogical intentions of the plan of simulation are that students structure their simulations, reflect about the simulations, and document their simulations.

Step	Probabilistic concepts	Fathom objects & operations
1	Construct the model, the random experiment	Choose type of simulation; define a (randomly generated) collection, simulate the random experiment
2	Identify events and random variables of interest (*Events and random variables as bridging concepts*)	Express events and random variables as “measures” of the collection
3	Repeat the model experiment and collect data on events and random variables	Collect measures and generate a new collection with values of the measures
4	Analyze data: relative frequency (events); empirical distribution (random variables)	Use Fathom as a data analysis software

Table 1: Four-step-design as a guideline for stochastic modeling

Our hypothesis is that the simulation plan is a helpful metacognitive tool for students and that Fathom is supportive of developing fluent simulation competence.

In the first step of the simulation plan one has to choose the type of simulation. We distinguish three different types of simulation in Fathom: the *simultaneous simulation*, in which the single experiment corresponds to different columns, the *sequential simulation*, in which the single experiment corresponds to different rows, and a *simulation as a sampling from an urn* (Maxara, 2006).

The four-step-design will be exemplified now on a typical task for the students. Thereby we demonstrate a perfect realization on the one hand and discuss possible problems on the other hand.

The problem: Mister Becker has to wear a black suit during his working hours, but he can choose the tie himself. 7 different ties hang in his wardrobe. Every morning he randomly takes one tie out of the wardrobe and puts it back in the evening.

1. What is the probability that Mister Becker will wear 5 different ties in his five-day working week?
2. What is the probability that Mister Becker will wear at least two identical ties in his five-day working week?
3. How many different ties does Mister Becker wear on average in his five-day working week?

Tasks:

a) Formulate the situation as a compound random experiment and specify the sample space W and the probabilistic assumptions.
b) Provide a plan of simulation and estimate the unknown probabilities of 1., 2., and 3 with a Fathom-simulation.
c) Document your work in a Fathom file.

Stage 1: Defining the random experiment

Perfect realization: On the probabilistic side the students have to make the following assumptions: each tie has the same probability to be drawn and the samples are stochastically independent. The model could be a sampling from an urn that contains seven different balls, and five balls are successively sampled with replacement. In Fathom they have to choose an appropriate type of simulation. In this case this could be the sequential simulation, or sampling from an urn. We choose the sequential simulation. There we have to define a collection (e.g. named "week") and an attribute "tie" with the random machine "randomInteger(1;7)" or "randomPick(1;2;3;4;5;6;7)". Finally, we have to add five cases to the collection. Fathom is a supportive tool because it offers different random machines that resemble to concrete models.

Possible problems: During the construction of a simulation several decisions have to be taken. Each decision one has to take is a potential source of error with for a larger or smaller impact on the simulation and the interpretation of the simulation. In our study we could identify the following two problems: The first problem area is the transformation of the random experiment into a correct simulation in Fathom. This problem depends on decisions one, four and five. The second problem is the naming of the objects (decisions two and three). The decisions of naming are important as the names of objects are relevant for the interpretation of results at a later stage.

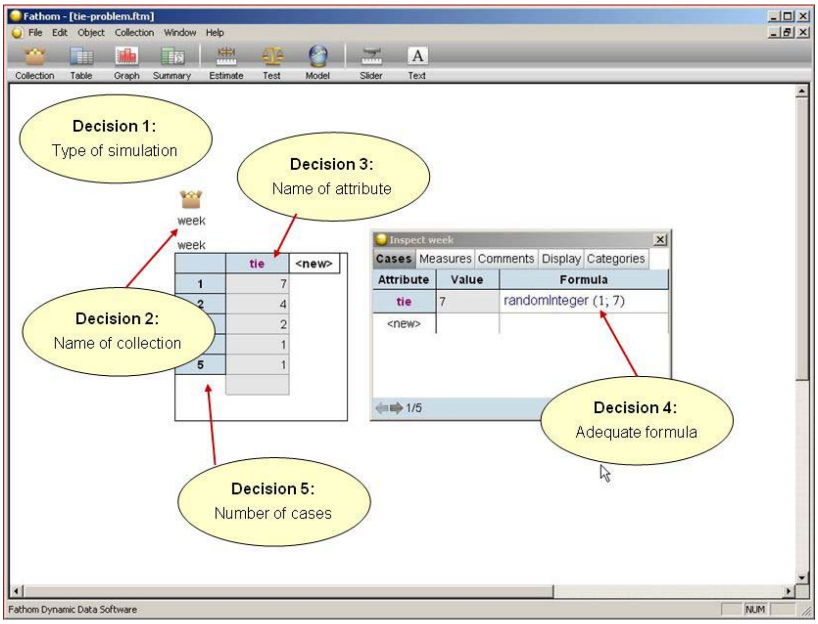

Figure 1: Sequential simulation of the tie-problem with 5 decision points

Stage 2: Events and random variables - defining measures

Perfect realization: In the second step, events and random variables should be identified and defined. On the probabilistic side they should be verbalized. For this example, we can define the two events by E1: "Mr. Becker wears five different ties in a week", E2: "Mr. Becker wears at least two equal ties in a week", and for the expected value we have to define the random variable X: "Number of different ties in a week". In Fathom, events and random variables correspond to *measures* that refer to the collection as a whole. In the collection, we have to designate and define three measures with adequate formulas: For E1: "five_different_ties": *uniqueValues(tie)=5*, for E2: "least_2_equal_ties": *uniqueValues(tie)* $\leq$ *4*, and for X: "number_different_ties": *uniqueValues(tie)*. Fathom is supportive in two respects: the concept of "measure" is a natural representation of an event or random variable, and second, commands such as "uniqueValues" makes this typical type of analysis direct and easy.

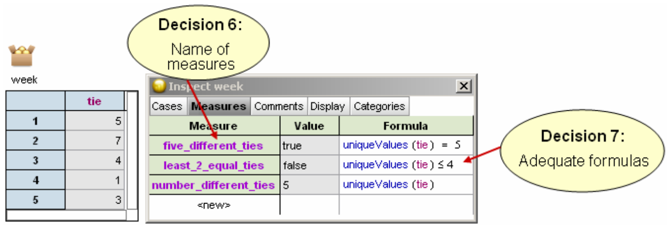

Figure 2: Defining measures

Possible problems: The problems in this stage are the naming of the measures (decision six) and the knowledge and implementation of correct formulas (decision seven). Another identified problem is that students omit the description and distinction of events and random variables before defining measures, and they try to transform (only) their colloquial verbalization into measures.

Stage 3: Repeating the (compound) experiment - collecting measures

Perfect realization: The model experiment has to be repeated. Thereby we have to collect data about events and random variables and to decide on the number of repetitions. In Fathom, this corresponds to collecting *measures*. If you collect *measures* in Fathom a new collection of measures is provided with five automatic repetitions. Now, you have to collect as many measures as you think are adequate for approximately estimating probabilities or distributions. In this example, the simulation was repeated 1000 times.

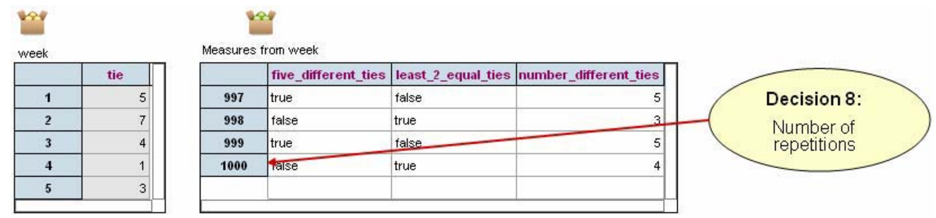

Figure 3: Collecting measures - repeating the simulation

Possible Problems: One problem field is the decision of the number *n* of repetitions. Are students reflecting about *n* or do they always take 1000 repetitions by standard practice?

Stage 4: Data analysis

Perfect realization: The fourth step consists of data analysis. The probabilities that were asked for in questions one and two can be estimated by the relative frequencies of the respective event. The expected value can be estimated by the empirical mean of the random variable. In Fathom, we can use summary tables to calculate the empirical values by the formulas "columnproportion" for the relative frequencies of the events and "mean( )" for the mean of the random variable. The graphics (bar chart and histogram) can be used to visualize the data, but also to read off the values by dragging the cursor on the bars.

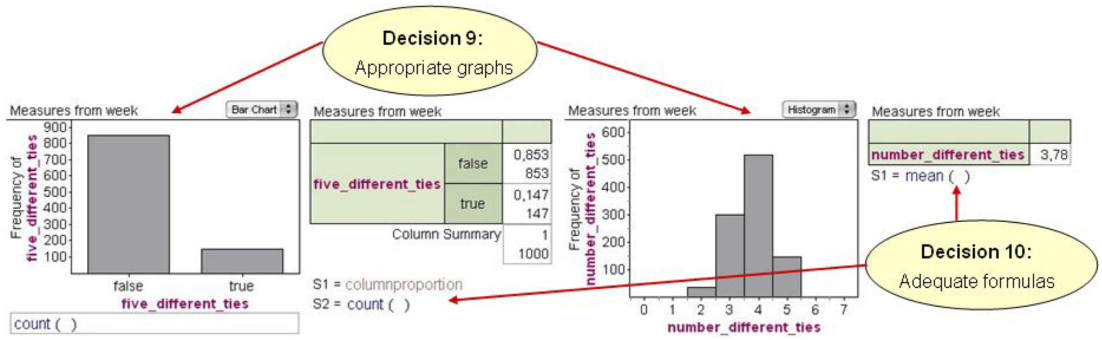

Figure 4: Data analysis in Fathom

Possible Problems: One source of problem is how to technically find the distribution and relative frequencies with Fathom. Another type of problem is related to interpreting the computed values. For instance, a mean of 3.78 could be rounded to 4 (ties per week), showing by this a misunderstanding of means and expected values.

STUDENTS' COMPETENCIES AND DIFFICULTIES DURING THEIR SIMULATION ACTIVITIES

The students have to have different stochastic and Fathom competencies to cope with the simulation in Fathom and the possible problems they might encounter. Our working group distinguished four Fathom competencies: *general Fathom competence*, *formula competence*, *simulation competence* and *strategic and generalizing competencies* (see Keitzer (2006) for a first application of these concepts). The general Fathom competence contains the knowledge of tools and their basic functionality in Fathom as well as the handling of the objects on the screen (clear design). The handling of the formula editor and the knowledge of formulas in different contexts were integrated into formula competencies. The simulation competence describes the competence to transform the random experiment into a Fathom simulation and to give meaningful naming of objects. The strategic and generalizing competencies catch the handling and avoidance of errors.

We have used these Fathom competencies and the introduced schema of the simulation procedure to analyze the problem solving activities of the eight students. The possible problems mentioned in each stage of simulation needed different Fathom competencies to be handled: Problems of transformation require simulation and formula competence, problems of naming are part of simulation competencies, and knowledge about formulas is integrated into the formula competence. But not all problems are only connected with Fathom competencies, they also involve probabilistic competencies, reflection and the ability to connect the two sides. Those problems are for instance, the definition and distinction of events and random variables and the interpretation of the computed results. Below, we illustrate some exemplary competencies and difficulties of the students.

Exemplary competencies

All pairs were able to solve the task through a simulation in Fathom and obtained correct results. Students' competencies were found in all four distinguished Fathom competencies and also on the probabilistic side.

Simulation competence: To illustrate the simulation competence we look at the following part of transcript.

1 S 1: I shall now, well, we have chosen the sampling simulation.
2 S 2: Mmh.
3 S 1: We have to put seven ties into the collection.
4 S 2: Mmh.
5 S 1: And then I would take a sample …
6 S 2: Mmh.

7 S 1: … of size five.
8 S 2: Mmh.
9 S 1: Wouldn't I? Because it refers always to one week.

Student 1 explains to the other student what to do next and what the Fathom steps mean in terms of the random experiment. She has a reflective view on their working and simulation process and makes up a plan of the following simulation steps.

Formula competence: The following example will show a good formula competence. Right at the beginning to simulate the random experiment the students have created a collection with an attribute.

10 S 3: And these are the ties. (*Names the attribute "ties"*)
11 S 4: Mmh.
12 S 3: And these are -, how many are in there?
13 S 4: Of what? We have seven ties, haven't we? (*Opens the formula editor*) RandomInteger. (*Type the formula*)
14 S 3: One comma seven.

The students do not talk about which formula they have to use, one of them stating the correct formula and the other completing it with the required values. It seems that they have adapted the random machines as representations of models for random experiments and do not have to think much about an adequate formula.

Stochastic competence: The following example shows students' understanding of the concept of the law of large numbers. A pair talked about the approximation of the relative frequencies of the measures to the theoretical probabilities and wrote into their document: "One can regard the relative frequencies of the measures as an approximation for the probabilities. The probability that Mister Becker wears five different ties is approximately 16.6%." By this sentence the students show an understanding of the estimation aspect, the fact that the simulated result is an approximation of the theoretical probability and not the probability itself.

Exemplary difficulties

We identified several domains of difficulties during the four stages: difficulties in transforming the random experiment into a simulation, lacks in formula competence and difficulties pertaining to the probabilistic part and the connection to the Fathom simulation. Here are only some examples for the observed difficulties.

Problem: Omitting probabilistic steps After defining the random experiment as a simulation, students omit the description and distinction of events and random variables and transform their colloquial verbalization or the task itself into a measure. In this case the students have created the simulation of the random experiment without discussion about what events or random variables to define as measures.

32 S 3: And now we should define measures. How do we call the first measures? (*She opens the inspector of the collection*). Anyway we have more then one. (*Naming the first measure "E1"*)

33 S 4: Mmh. (*S3 types the formula uniqueValues(tie)=5*)
34 S 4: Yes?
35 S 3: Yes.

The naming of the measure indicates that the students do not think about the mathematical type of object. The reification of event and random variable in Fathom is not strengthened as intended in the theoretical part of the course. These two concepts are rather blurred in students' minds, probably because both of them were defined as measures. The students use the Fathom-concept of measures as a replacement for the stochastic concepts of event and random variable in an efficient way. This shows a need for additional theoretical concept building.

Difficulty: Interpreting the results Difficulties in interpreting simulated results are related to a still unsatisfactory comprehension of the difference between relative frequency and probability. The simulated relative frequency of 0,171 is interpreted (at least in the text) as the (definite) probability to use five different ties per week. Here the students do not clearly distinguish consciously between those two concepts.

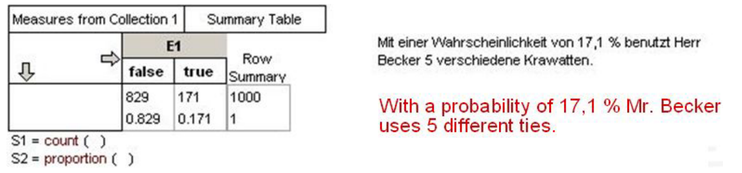

Figure 5: Exemplary interpretation of a simulated result

SUMMARY AND CONSEQUENCES

Despite some difficulties, the students of our study were able to simulate these kinds of stochastic situations in Fathom. They acquired the intended software competence and accepted the plan of simulation as guidance. We have distinguished four Fathom competencies that the students should have to simulate such kinds of random experiments. The general Fathom competence was acquired by all students. The other three competencies were acquired in different depths like the examples illustrate.

The substitution of mathematical competence by simulation competence is not difficult for the students. But most students do not use this freedom to concentrate on the probabilistic concepts behind the simulation. The reification of event and random variable as different kinds of measures did not work as expected. These students do not distinguish explicitly between events and random variables during their simulation process, but they have a working knowledge about them. The concepts tend to remain separated in the two worlds, the "World of probability" and the "World of Fathom". Thus – as an instructional implication of the study – the informal use of these two concepts should be more deeply and explicitly related to each other in the course.

As a consequence it looks expedient to put more emphasis on the explanatory aspect of the simulation plan, so as not to only provide technical help, but also to foster students' language, knowledge and reflection to link simulation aspects with probabilistic concepts. Students should reflect on their simulation activities, about the basic probabilistic concepts and their relation to the simulation. Another aspect is to support a closer relation between the "World of probability" and the "World of Fathom" through more comparisons of simulations and theoretical mathematical solutions.

The conclusions of this paper are also relevant to other countries than Germany in different aspects, because both the understanding of probability concepts and the simulation of random experiments are essential in teaching statistics. The use of technology is an important aspect in teaching and learning statistics. We think that our modelling guideline for simulation – setting up a stochastic model, writing a plan of simulation and realizing the plan in Fathom – is adaptable (perhaps with some modifications in relation to the range of random experiments) for other simulation-software. The general categories of students' competencies and difficulties are also applicable to other software, thus they could be of general interest to statistics instructors and researchers.

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