\[ \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \]

\[ e^x = \lim_{n \to \infty} \left( 1 + \frac{x}{n} \right)^n \]

\[ t \frac{\sin(t)}{t} = \begin{cases} \cos(t) & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases} \]

\[ \int_0^\infty e^{-x} \, dx = 1 \]

\[ \frac{d}{dx} \log(x) = \frac{1}{x} \]

\[ \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \]

**Theorem:** If \( f \) is a continuous function on \( [a, b] \), then for any \( c \in (a, b) \),

\[ \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \]

**Proof:**

Let \( f \) be a continuous function on \( [a, b] \), and let \( c \in (a, b) \). By the Fundamental Theorem of Calculus, for any \( x \in [a, b] \),

\[ F(x) = \int_a^x f(t) \, dt \]

is an antiderivative of \( f \). Then,

\[ F(c) = \int_a^c f(t) \, dt \]

and

\[ F(b) = \int_a^b f(t) \, dt \]

Therefore,

\[ \int_a^b f(x) \, dx = F(b) - F(a) = \int_a^c f(t) \, dt + \int_c^b f(t) \, dt \]

\[ \blacksquare \]

**Application:**

Consider the function \( f(x) = x^2 \) on \( [0, 2] \). Let \( c = 1 \), then

\[ \int_0^2 x^2 \, dx = \int_0^1 x^2 \, dx + \int_1^2 x^2 \, dx \]

\[ \left[ \frac{x^3}{3} \right]_0^2 = \left[ \frac{x^3}{3} \right]_0^1 + \left[ \frac{x^3}{3} \right]_1^2 \]

\[ \frac{8}{3} = \frac{1}{3} + \frac{8}{3} \]

\[ \frac{8}{3} = \frac{8}{3} \]

\[ \therefore \text{The statement is true.} \]

**Remark:** This theorem is a powerful tool in calculus, allowing us to break up integrals into smaller, more manageable pieces. It is widely used in various applications, including physics, engineering, and economics.